Lecture 20: Flux Shaping and Power Peaking (The "Hot Spot" Problem)

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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Introduction: The "Average" Trap

In previous lectures, we treated the reactor as a "Point" (k_{eff}) or a perfect "Sine Wave." In reality, a reactor core is a heterogeneous mix of fuel, poison, water, and steel.

The Problem: The safety of a reactor is limited by its **hottest** fuel pin, not its average pin.

- If the reactor is producing 3000 MW total, but one region is producing 5x the average power density, that region will melt.
- We define the **Peaking Factor** (F_q):

$$F_q = \frac{\text{Maximum Power Density}}{\text{Average Power Density}}$$

Goal: The goal of reactor design is to make F_q as close to 1.0 as possible (i.e., "Flatten the Flux").

1 The Natural Shape: The Cosine

For a uniform, bare cylindrical reactor, diffusion theory dictates the flux shape is a Bessel function (radial) and Cosine (axial):

$$\phi(r, z) = \phi_{max} J_0 \left(\frac{2.405r}{R} \right) \cos \left(\frac{\pi z}{H} \right)$$

The Peak-to-Average ratio for this "natural" shape is:

$$\Omega \approx 3.64$$

This is terrible! It means the center of the core is working nearly 4 times harder than the edges. We are wasting the capacity of the fuel at the periphery.

2 Distortion by Control Rods (Flux Tilting)

As noted in class, inserting control rods distorts this shape further.

- **Partial Insertion:** If we insert a rod bank halfway ($z = H/2$), the flux is pushed to the bottom of the core. The peak shifts and intensifies.

- **Asymmetry:** If we insert rods on the left but not the right, the flux "tilts" to the right.

This creates local zones where $k_\infty > 1$ feeding zones where $k_\infty < 1$. The reactor holds together only because of neutron diffusion.

3 Engineering Solutions: Flattening the Flux

How do we force the curve to be flat?

3.1 1. Reflector Savings

Surrounding the core with a reflector (water, graphite, or beryllium) is the most fundamental way to improve neutron economy and flatten the power distribution.

3.1.1 How a Reflector Works

A reflector is not a "mirror" in the optical sense. It works via **diffusive scattering**.

- Neutrons leak out of the fuel region into the reflector material.
- Instead of escaping to the environment, they collide with the reflector nuclei.
- The reflector acts as a "random walk" generator. A significant fraction of these neutrons (measured by the **Albedo** or reflection coefficient, β) scatter back into the core.
- **The Result:** The neutron flux at the core boundary (R) is no longer zero. It is raised to some finite value.
- Mathematically, this makes the core "feel" like it is larger than it physically is. We call this effective increase the **Reflector Savings** (δ).

  $$R_{bare} = R_{physical} + \delta$$

  Because the flux does not have to drop to zero at the physical edge, the cosine shape in the center is flattened, and F_q drops from 3.64 to ≈ 2.5.

3.1.2 Material Selection: Beryllium vs. Water

A good reflector must have a high scattering cross-section (Σ_s) and a low absorption cross-section (Σ_a).

- **Water (H_2O):** Excellent scatterer (hydrogen is light), but has moderate absorption. It is cheap and doubles as coolant.
- **Graphite (C):** Lower scattering per cm than water, but extremely low absorption. Requires thick layers (meters) to be effective.
- **Beryllium (Be):** The "Gold Standard" of reflectors.
  1. **Mass:** Low atomic mass (9 amu) means high energy loss per collision (good thermalizer).
  2. **Absorption:** Very low Σ_a.

3. **The $(n, 2n)$ Reaction:** Unlike water or carbon, Beryllium can multiply neutrons. If a high-energy neutron (> 1.67 MeV) hits Be-9, it can trigger:

$$^{9}Be + n \rightarrow 2\alpha + 2n$$

This essentially gives you "bonus" neutrons, effectively making the Albedo > 1 for those specific energies.

Note: Beryllium is toxic and expensive, so it is usually reserved for research reactors or weapons, not commercial power plants.

Historical Case Study: The "Demon Core" (1945–1946)

The power of reflectors is best illustrated by two fatal accidents at Los Alamos involving the same 6.2 kg sphere of Plutonium-239, nicknamed the "Demon Core."

- **The Physics:** The core was subcritical ($k_{eff} < 1$) in air. It relied on neutron leakage to stay safe.
- **Incident 1 (Harry Daghlian, 1945):** He was manually stacking tungsten carbide bricks (a neutron reflector) around the core. He accidentally dropped a brick onto the assembly. The sudden increase in reflection shut off the leakage path, k_{eff} spiked above 1.0, and he received a lethal dose.
- **Incident 2 (Louis Slotin, 1946):** Slotin was lowering a **Beryllium** hemispherical shell over the core, holding it apart with the tip of a flathead screwdriver. The screwdriver slipped. The Be shell closed completely.
- **The Result:** The Beryllium reflected reflected a very large fraction of leaking neutrons back into the Pu. The core went **Prompt Critical** instantly, creating a flash of blue light (Cherenkov radiation in the air and the fluid in his eyes). Slotin died 9 days later.

The Warning: Several months prior to the accident, Enrico Fermi had warned Slotin about his manual technique: *"Keep doing that experiment that way and you'll be dead within a year."* Slotin died in May 1946.

3.2 2. Chemical Shim (Soluble Boron)

This is the primary method for PWRs. Instead of inserting control rods (which poke "holes" in the flux), we dissolve Boric Acid in the coolant.

- The poison is distributed uniformly in the liquid.
- It depresses the flux everywhere equally.
- **Benefit:** We can run with control rods fully withdrawn ("All Rods Out" - ARO), preserving the natural, smoother cosine shape rather than a distorted one.

3.3 3. Zone Loading (Enrichment Zoning)

We load different fuel into different parts of the core.

- **Center:** Load **Lower** Enrichment (e.g., 2.0%).
- **Periphery:** Load **Higher** Enrichment (e.g., 4.5%).

The high enrichment at the edges boosts the power in the "lazy" regions, while the low enrichment in the center suppresses the peak. This flattens the curve significantly.

3.4 4. Burnable Poisons (Gadolinium)

We mix neutron poisons (Gadolinium or Erbium) directly into the fuel pellets, but only in the freshest, most reactive fuel assemblies.

- As the reactor runs, the Gadolinium "burns out" (transmutes to low cross-section isotopes).
- This compensates for the loss of Uranium.
- It keeps the local power peaks suppressed early in life.

4 The Consequence of Size: Spatial Instability

If a reactor is physically very large (specifically, large compared to the neutron **Migration Length**, M), the regions become "decoupled."

- A perturbation in the top half may not be "felt" by the bottom half immediately.
- This can lead to **Xenon Oscillations**: Power rises in the top → Xenon builds up → Power drops in top → Power shifts to bottom → Xenon burns out in top → Power swings back.
- Operators effectively end up "chasing" the power bubble around the core.

The Chernobyl Connection (RBMK Design)
The RBMK-1000 was a massive reactor (7m high, 12m diameter). Its large graphite moderator made the Migration Length short relative to the core size. It was highly susceptible to "decoupled" spatial power instabilities, requiring complex automatic control systems that were disabled on the night of the accident.

5 Summary

- F_q **(Peaking Factor):** The ratio of Max to Average power. Limits total reactor output.
- **Reflectors:** Scatter neutrons back to the core, raising edge flux and flattening the center. Be ¿ C ¿ H2O in terms of physics efficiency.
- **Chemical Shim:** Uses uniform poison (Boron) to minimize rod insertion and keep flux flat.
- **Zone Loading:** Putting higher octane fuel at the edges to utilize the whole core volume efficiently.